



# MECHANICS

## Lecture No.3 Friction

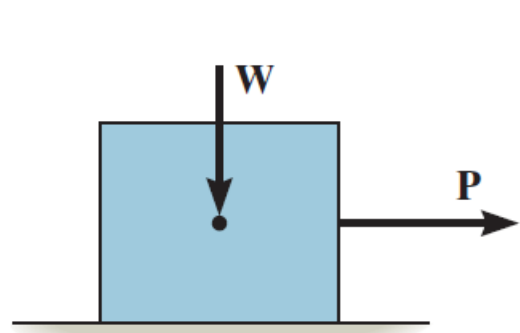
د. محمد سعد



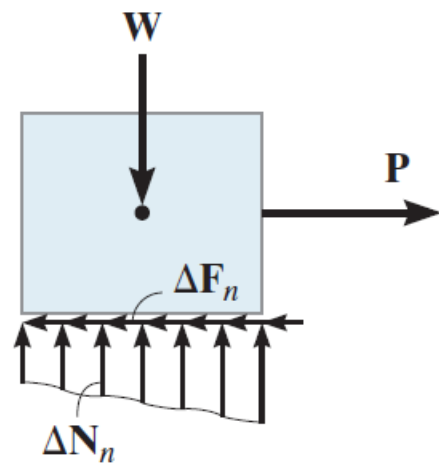
# Friction

*Friction* is a force that resists the movement of two contacting surfaces that slide relative to one another. This force always acts *tangent* to the surface at the points of contact and is directed so as to oppose the possible or existing motion between the surfaces.

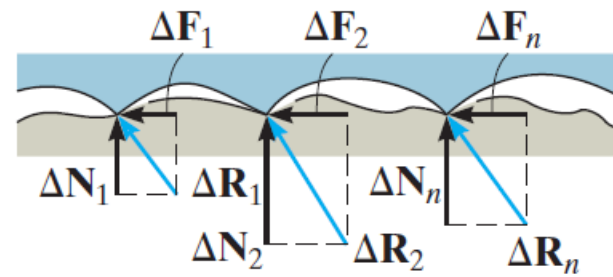
In this lecture, we will study the effects of *dry friction*, which is sometimes called *Coulomb friction* since its characteristics were studied extensively by the French physicist Charles-Augustin de Coulomb in 1781. Dry friction occurs between the contacting surfaces of bodies when there is no lubricating fluid.



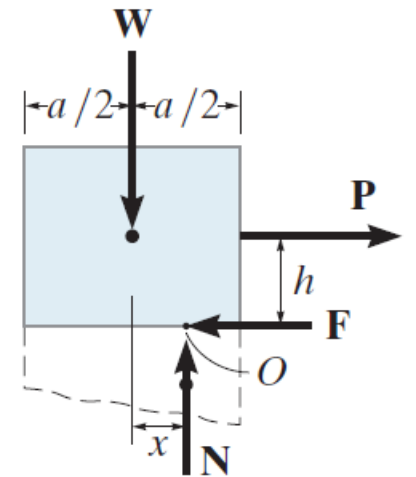
(a)



(b)



(c)

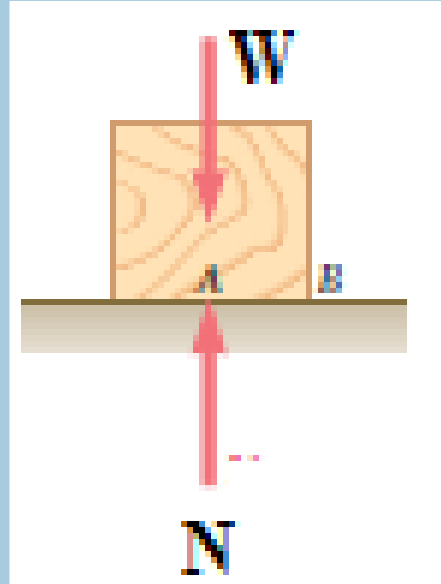


Resultant normal and frictional forces

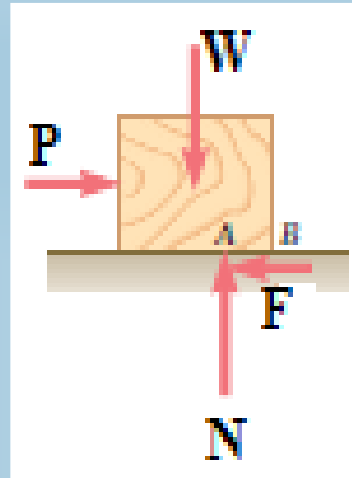
(d)

# The laws of friction are exemplified by the following experiment

1. A block of weight  $W$  is placed on a horizontal plane surface.



2. A horizontal force  $P$  is applied to the block. If  $P$  is small, the block will not move; some other horizontal force must therefore, which balances  $P$ . This other force is the static **friction force**  $f$



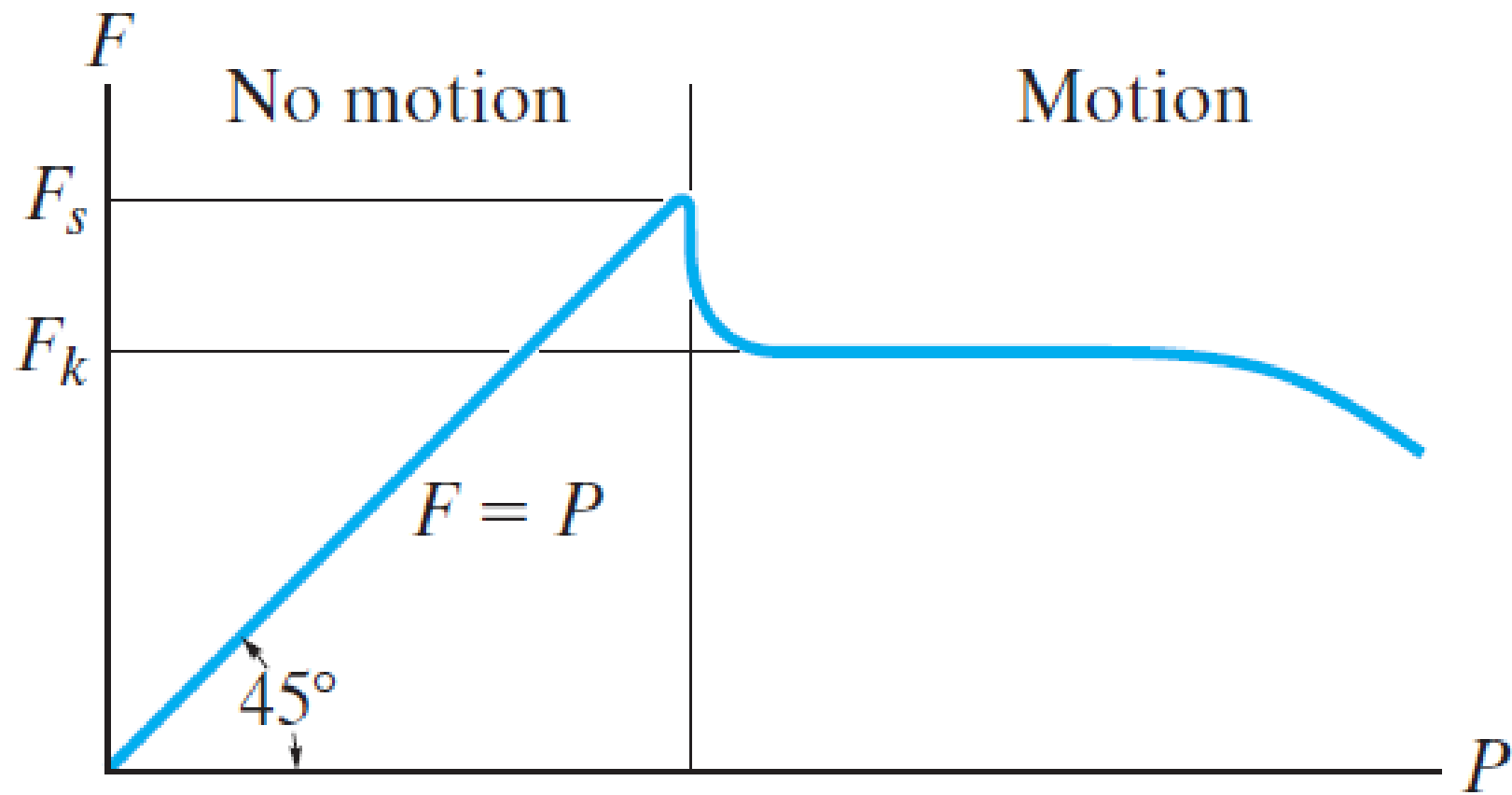
3. If the force  $\mathbf{P}$  is increased, the friction force  $\mathbf{F}$  also increases, continuing to oppose  $\mathbf{P}$ , until its magnitude reaches a certain maximum value  $\mathbf{F}_m$

4. If  $\mathbf{P}$  is further increased, the friction force cannot balance it anymore and the block starts sliding.

**:NOTE**

If  $\mathbf{N}$  reaches the point  $\mathbf{B}$  before  $\mathbf{F}$  reaches its maximum value, the block will tip about  $\mathbf{B}$  before it can start sliding

5. As soon as the block has been set in motion, the magnitude of  $\mathbf{F}$  drops from  $\mathbf{F}_m$  to a lower value  $\mathbf{F}_k$  (kinetic friction force)



Then from the previous experiment;

The value  $F_m$  of the static friction force is proportional to the normal component  $N$

$$F_m = \mu_s N$$

Where

**$\mu_s$  is the coefficient of static friction**

The magnitude of  $F_k$  the kinetic friction force may be put in the form

$$F_k = \mu_k N$$

Where

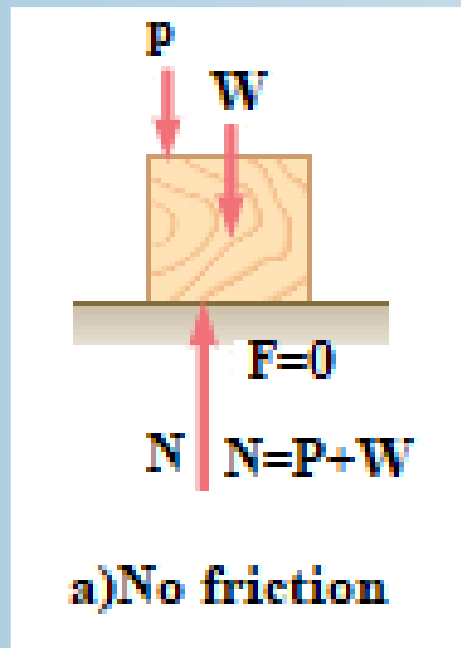
**$\mu_k$  is the coefficient of kinetic friction**

# NOTES

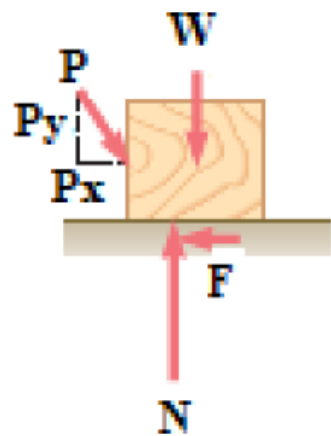
1. The maximum frictional force  $F$  is proportional to the normal force  $N$ .
2. The limiting static friction force is greater than the kinetic frictional force.

From above, there are four different situations can occur when a rigid body is in contact with a horizontal surface:

1. The forces applied to the body do not tend to move it along the surface of contact; there is no friction force.



2. The applied forces tend to move the body along the surface of contact but are not large enough to set it in motion. The friction force  $F$  which has developed can be found by solving equation of equilibrium for the body. Since there is no evidence that  $F$  has reached its maximum value, the equation  $F_m = \mu_s N$  cannot be used to determine the friction force.



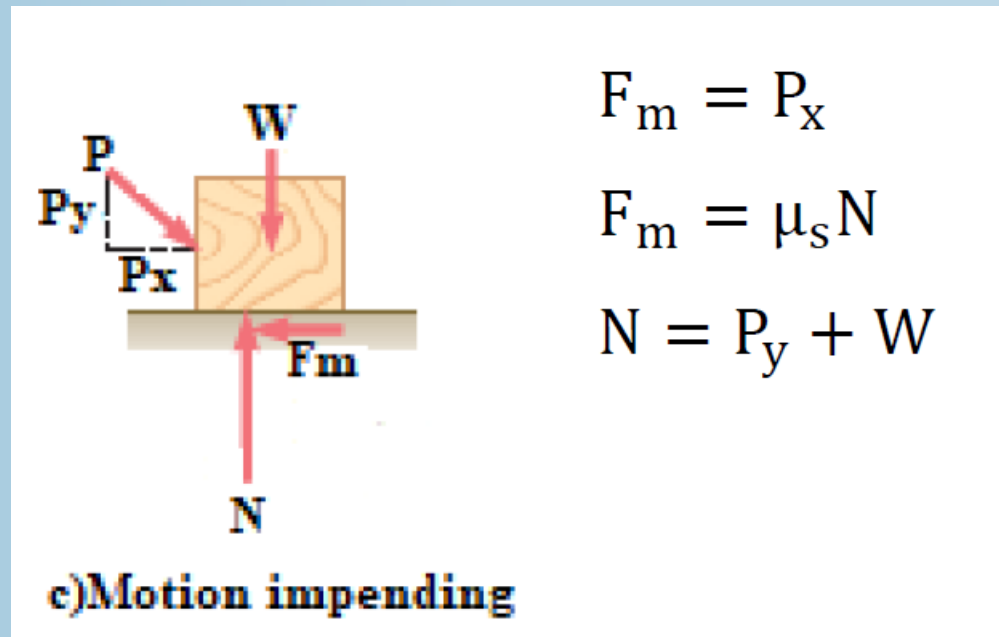
**b) No Motion**

$$F = P_x$$

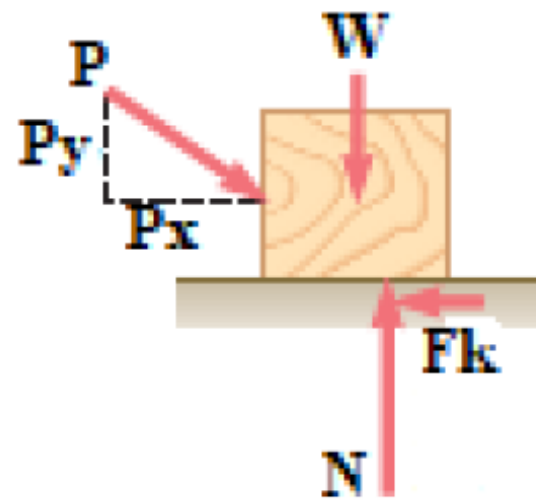
$$F < \mu_s N$$

$$N = P_y + W$$

3. The applied forces are such that the body is just about to slide. We say the motion is impending,  $F$ . The friction force  $F$  has reached its maximum value  $F_m$  and together with the normal force  $N$ , balances the applied force. Both the equations of equilibrium and the equation  $F_m = \mu_s N$  can be used.



The body is sliding under the action of the applied forces; and the equations of equilibrium do not apply any more. However,  $F$  is now equal to  $F_k$



$$F_k < P_x$$

$$F_k = \mu_k N$$

$$N = P_y + W$$

d) Motion

# Coefficient of Friction

The coefficient of static friction  $\mu_s$  is defined as the ratio of the magnitude of the maximum static frictional force  $F$ , to the magnitude of the normal force  $N$ , between the two surfaces. It is depend on the nature of the surfaces in contact.

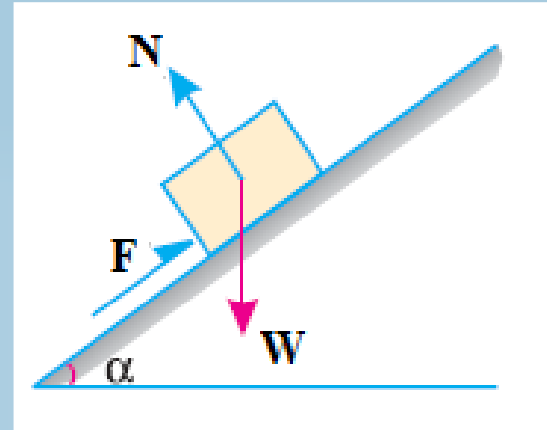
$$\mu_s = \frac{F_m}{N}$$

Approximate values of coefficient of static friction for various dry surfaces are given in the following table. The corresponding values of the coefficient of kinetic friction would be about 25 percent smaller.

Surface in contact	$\mu_s$
Steel on steel	0.4-0.8
Wood on wood	0.2-0.5
Metal on stone	0.3-0.7
Rubber on concrete	0.6-0.8

# Angle of Friction

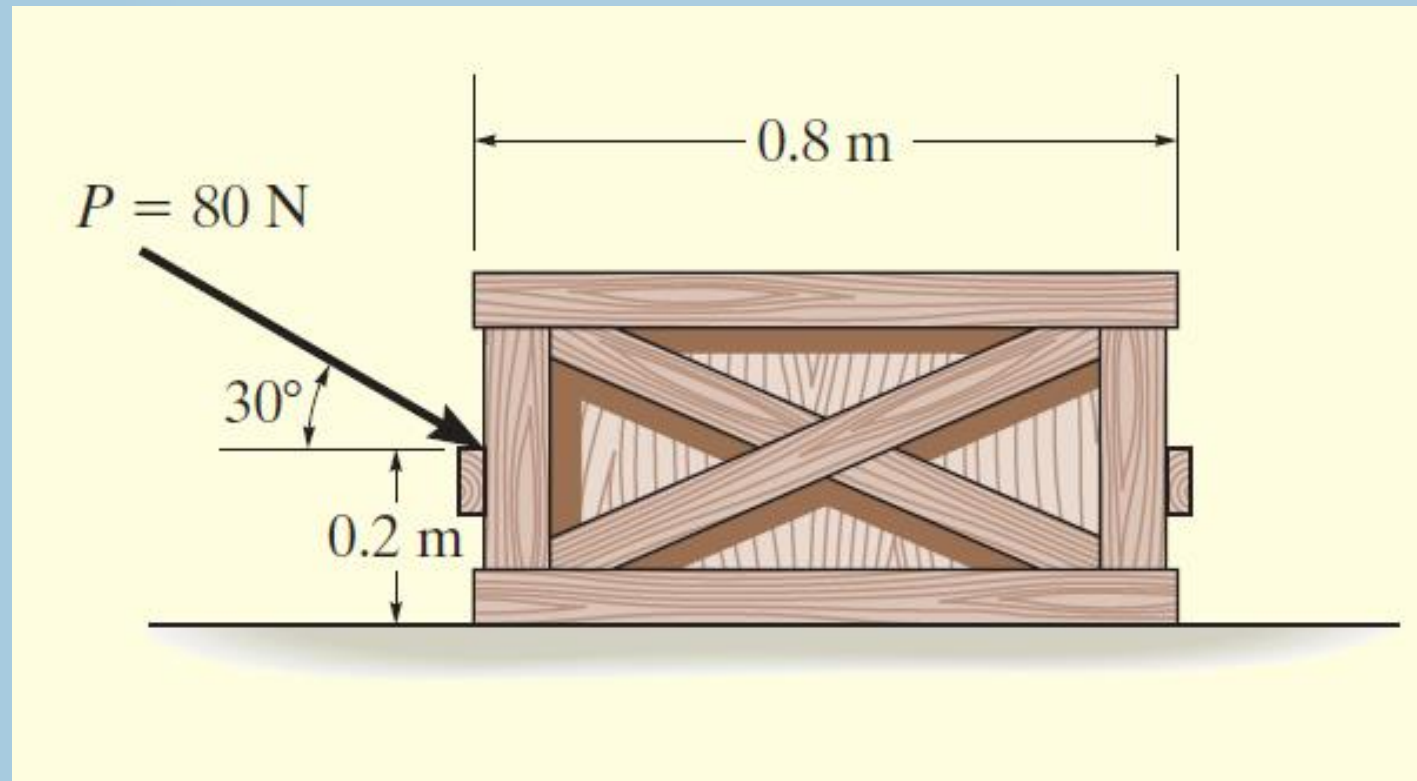
Consider a body of weight  $W$  resting on an inclined plane.



Let the angle of inclination ( $\alpha$ ) be gradually increased, till the body just start sliding down the plane. This angle of inclined plane ( $\phi$ ), at which a body just begins to slide down the plane, is called the angle of friction. This is also equal to the angle, which the normal reaction makes with the vertical.

$$\tan\phi = \frac{F}{N} = \mu_s$$

The uniform crate shown in figure has a mass of 20kg. If a force  $P=80\text{N}$  is applied to the crate, determine if it remains in equilibrium. The coefficient of friction is 0.3.



$$\rightarrow \sum F_x = 0$$

$$80 \cos 30 - F = 0 \quad F = 69.3 \text{ N } \leftarrow$$

$$\uparrow \sum F_y = 0$$

$$-80 \sin 30 + N_c - 196.2 = 0 \quad N_c = 236.2 \text{ N } \uparrow$$

$$\curvearrow \sum M_O = 0$$

$$80 \sin 30(0.4) - 80 \cos 30(0.2) + N_c(x) = 0$$

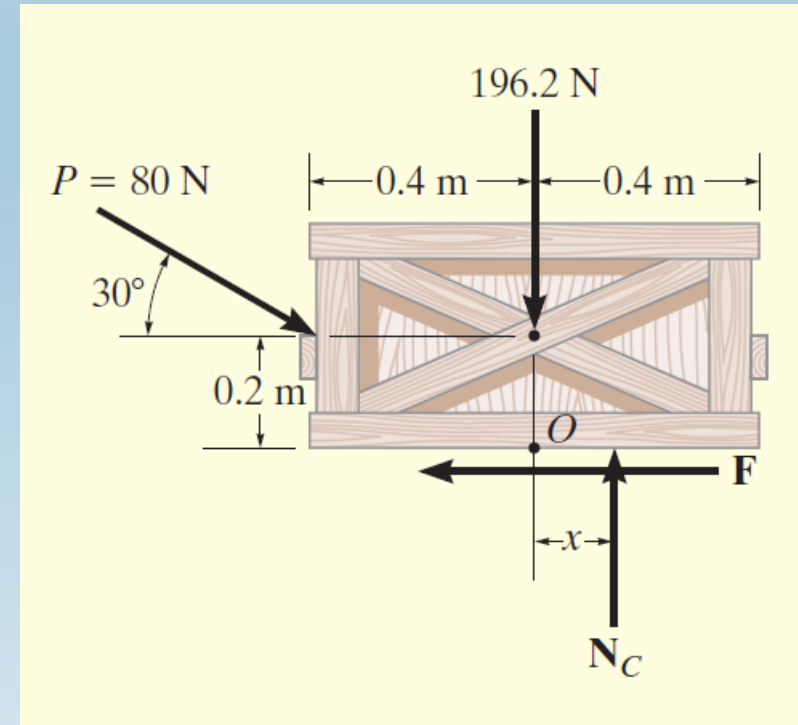
$$x = -0.00908 \text{ m} = -9.08 \text{ mm}$$

No tipping will occur since:-

1.  $x < 0.4 \text{ m}$

2.  $F = 69.3 < F_{\max} = \mu N_c = 0.3(236.2) = 70.9 \text{ N}$

$\therefore$  The crate was still in equilibrium.



It is observed that when the bed of the dump truck is raised to an angle of  $\theta=25^\circ$  the vending machines will begin to slide off the bed, determine the static coefficient of friction between a vending machine and the surface of the truck bed.



### **Solution:-**

From the F.B.D

$$\rightarrow \sum F_x = 0$$

$$W \sin 25 - F = 0 \quad (1)$$

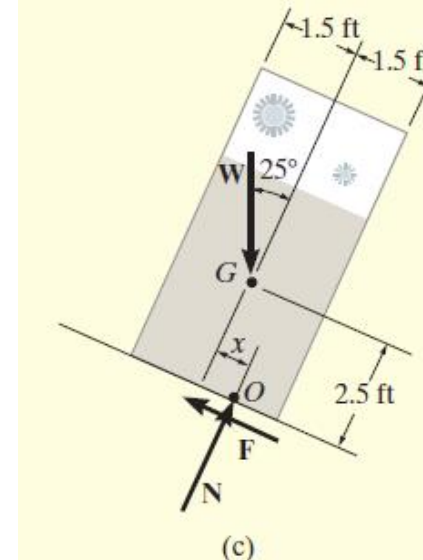
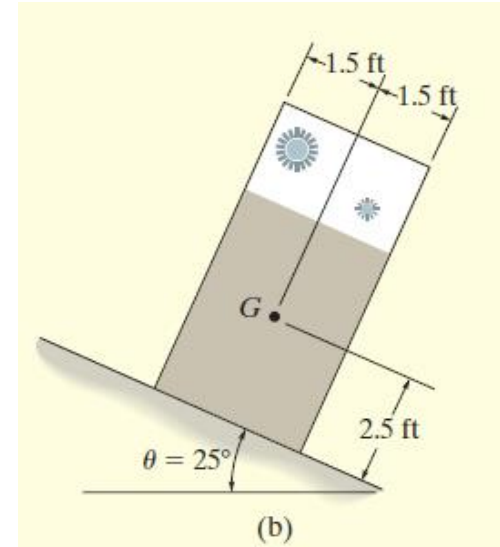
$$\uparrow \sum F_y = 0$$

$$N - W \cos 25 = 0 \quad (2)$$

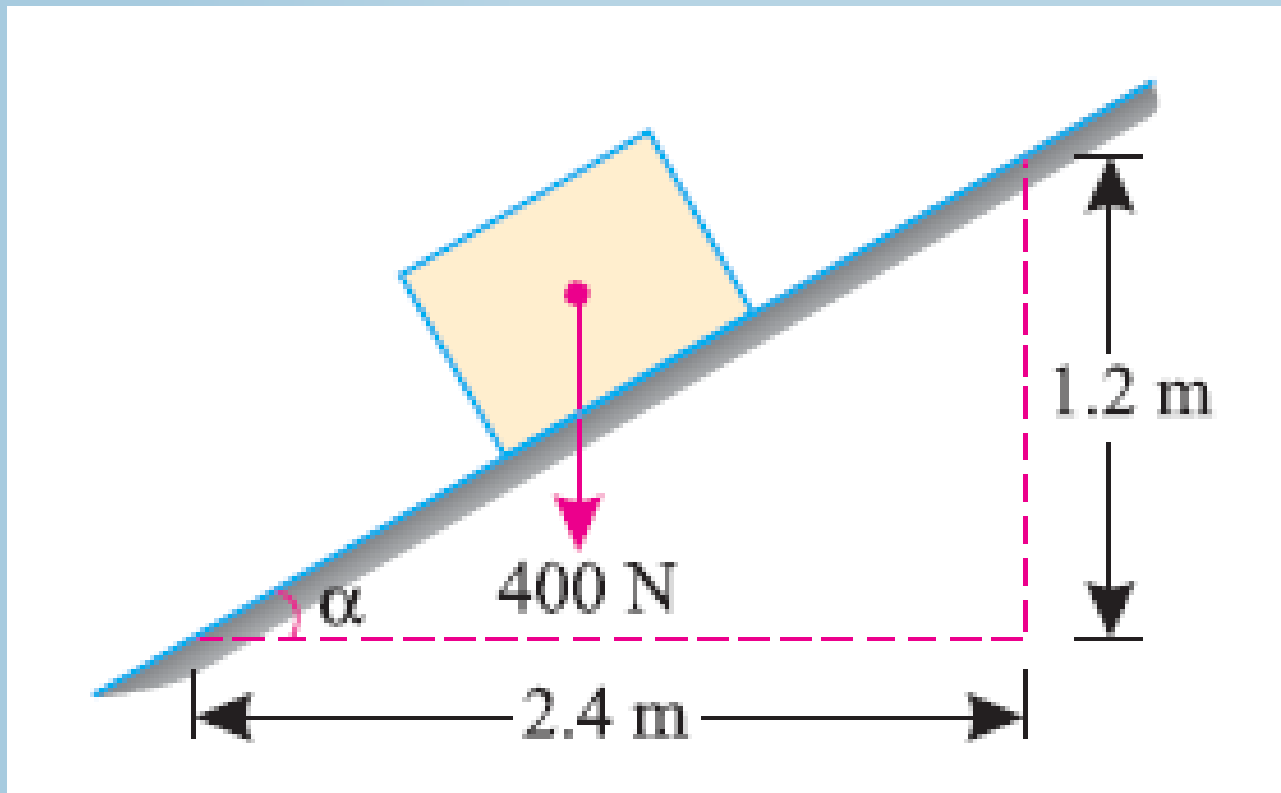
From Eqs (1) and (2)

$$F = \mu N \quad W \sin 25 = \mu(W \cos 25)$$

$$\mu = \tan 25 = 0.466$$



An inclined plane is used to unload slowly a body weighing **400N** from a truck **1.2m** high into the ground. The coefficient of friction between the underside of the body and the plank is **0.3**. State whether it is necessary to push the body down the plane or hold it back from sliding down. What minimum force is required parallel to the plane for this purpose.



### Solution:-

$$\tan \alpha = \frac{1.2}{2.4} = 0.5 \quad \alpha = 26.5^\circ$$

$$\underline{1. N = W \cos \alpha = 400 \cos 26.5^\circ = 357.9 \text{ N}}$$

$$F_m = \mu_s N = 0.3 \times 357.9 = \mathbf{107.3 \text{ N}}$$

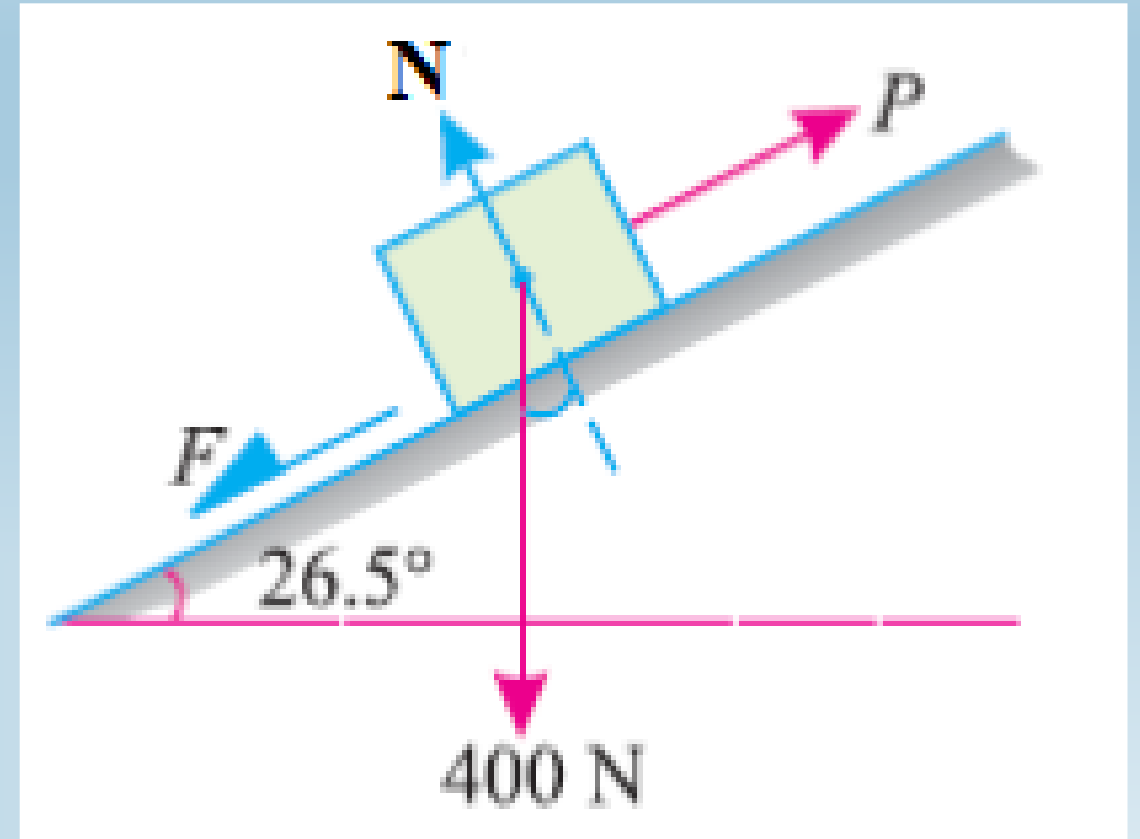
Resolving the 400 N along the plane.

$$= 400 \sin \alpha = 400 \times \sin 26.5^\circ = \mathbf{178.5 \text{ N}}$$

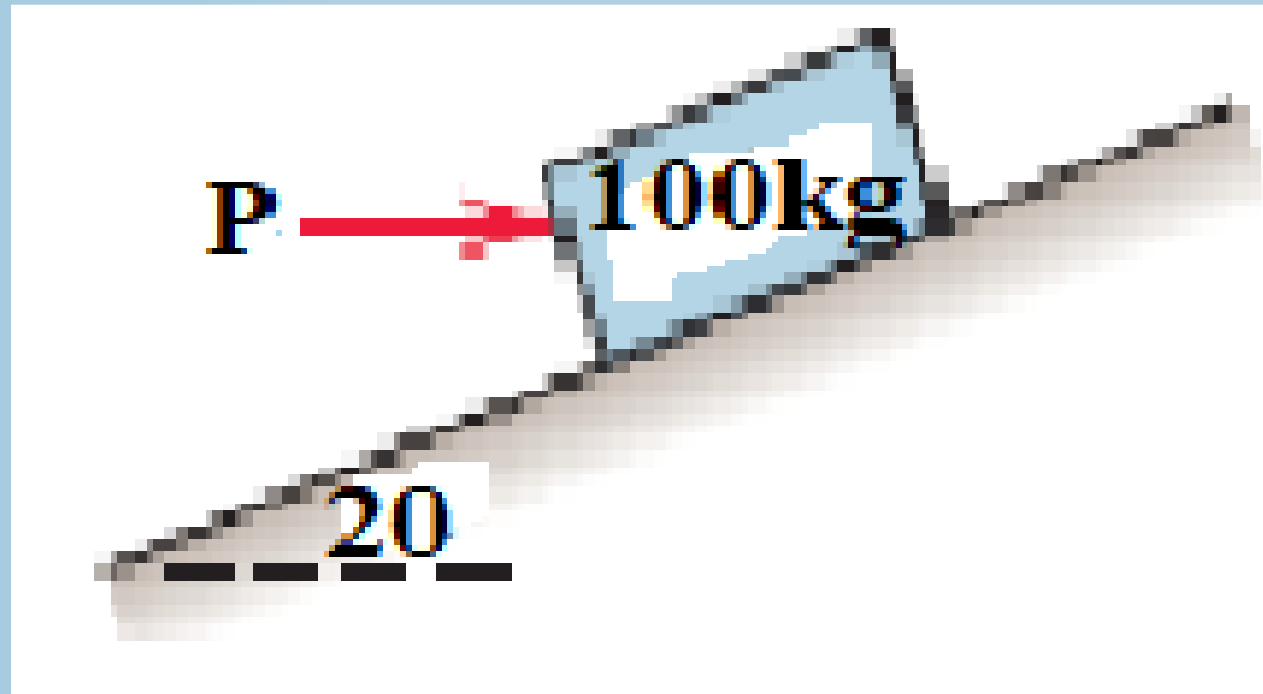
The force along the plane (which is responsible for sliding the body) is more than the force of friction; therefore, the body will slide down.

It is not necessary to push the body down the plane; rather it is necessary to hold it back from sliding down.

$$\underline{2. P = 178.5 - 107.3 = 71.2 \text{ N}}$$



Determine the magnitude and direction of the friction force acting on the 100kg block shown if, first,  $P=500\text{N}$  and, second,  $P=100\text{N}$ . The coefficient of static friction is 0.2, and the coefficient of kinetic friction is 0.17. The force is applied with the block initially at rest.



There is no way of telling from the statement of the problem whether the block will remain in equilibrium or whether it will begin to slip following the application of P, therefore assume the block is in equilibrium;

$$\rightarrow \sum F_x = 0$$

$$P \cos 20 + F - 981 \sin 20 = 0$$

$$\uparrow \sum F_y = 0$$

$$N - P \sin 20 - 981 \cos 20 = 0$$

1.  $P = 500 \text{ N}$  subs in above eqs  $F = -134.3 \text{ N}$   $N = 1093 \text{ N}$

$$F_{\max} = \mu N = 0.2(1093) = 219 \text{ N}$$

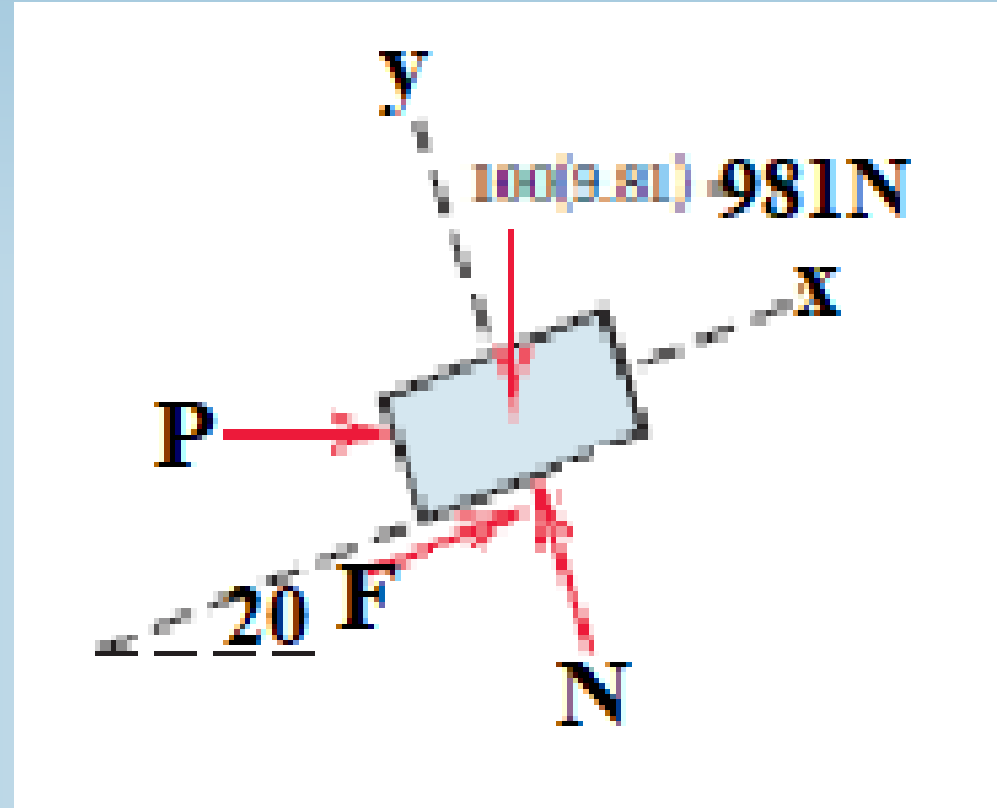
$F < F_{\max}$  then the assumption was correct  $F = 134.3 \text{ N}$   
down the plane

2.  $P = 100 \text{ N}$  subs in above eqs  $F = 242 \text{ N}$   $N = 956 \text{ N}$

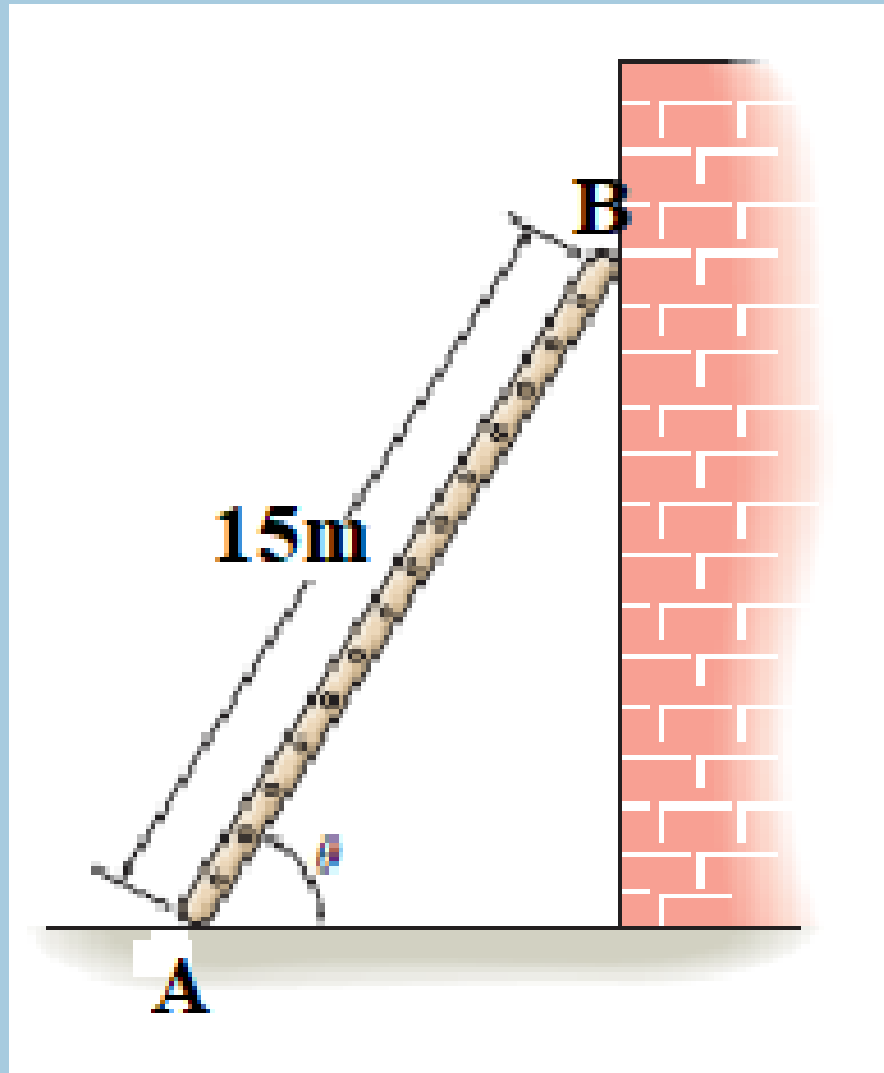
$$F_{\max} = \mu N = 0.2(956) = 191.2 \text{ N}$$

$F > F_{\max}$  then the assumption was incorrect

$$F_k = \mu_k N = 0.17(956) = 162.5 \text{ N up the plane}$$



The ladder has a uniform weight of 80N and rests against the wall at B. If the coefficient of friction at A and B is 0.4, determine the smallest angle  $\theta$  at which the ladder will not slip.



### Solution:-

Since the ladder is required to be on the verge to slide down, then:-

$$F_A = \mu N_A = 0.4N_A$$

$$F_B = \mu N_B = 0.4N_B$$

From the F.B.D.

$$\rightarrow \sum F_x = 0$$

$$0.4N_A - N_B = 0 \quad N_B = 0.4N_A \quad (1)$$

$$\uparrow \sum F_y = 0$$

$$N_A + 0.4N_B - 80 = 0 \quad (2)$$

Solving eqs.(1) and (2)

$$N_A = 68.97N \quad N_B = 27.59N$$

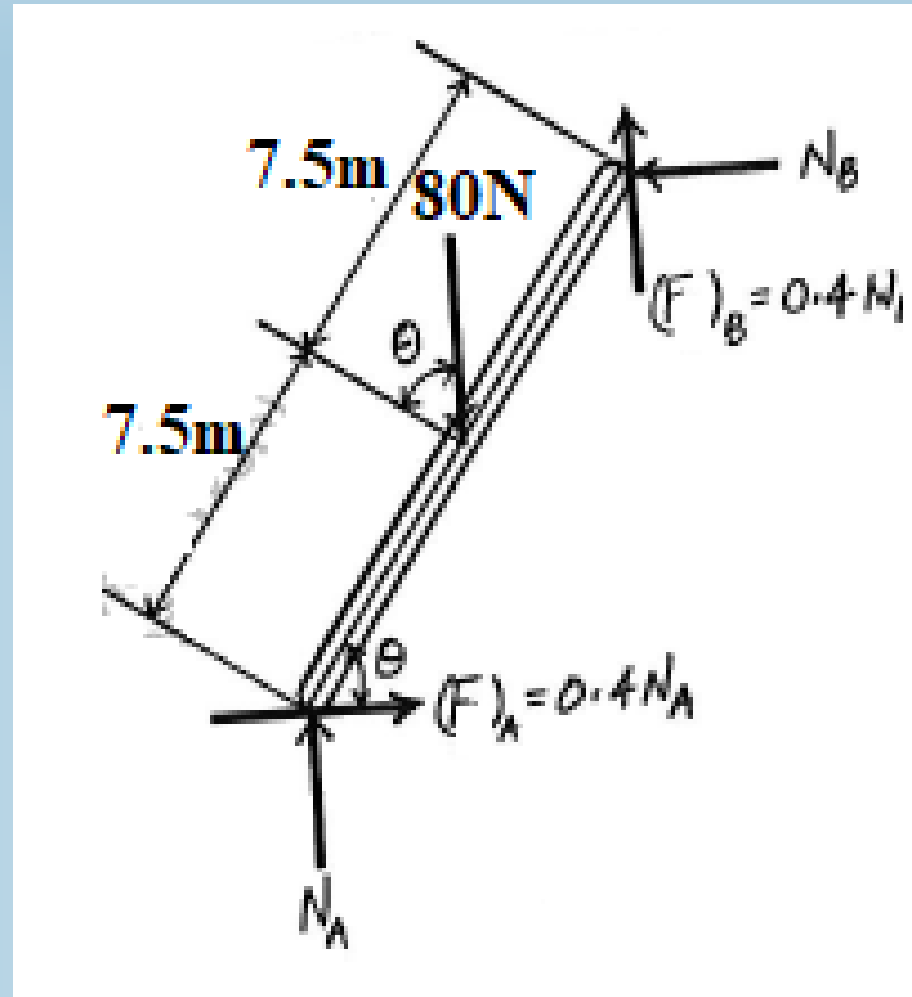
$$\curvearrow \sum M_A = 0$$

$$0.4(27.59)(15 \cos \theta)$$

$$+ 27.59(15 \sin \theta) - 80 \cos \theta(7.5) = 0$$

$$413.79 \sin \theta - 434.48 \cos \theta = 0$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{434.48}{413.79} = 1.05 \quad \theta = 46.4$$



# Thank you for listening

